# Low dimensional topology and number theory

March 17-20, 2009 Soft Research Park Center, Fukuoka

# Program

#### March 17

11:00 - 12:00

Susan Williams (University of South Alabama)

Twisted Alexander invariants and two-bridge knots

13:30 - 14:30

Yuichi Kabaya (Tokyo Institute of Technology)

A method for finding ideal points from an ideal triangulation and its application

14:50 - 15:50

Stavros Garoufalidis (Georgia Institute of Technology)

Asymptotics of classical spin networks

### March 18

10:00 - 11:00

Daniel Silver (University of South Alabama)

On a theorem of Burde and de Rham

11:20 - 12:20

Takahiro Kitayama (The University of Tokyo)

Torsion volume forms and twisted Alexander functions on character varieties of knots

14:00 - 15:00

Kazuo Habiro (RIMS, Kyoto University)

On the Witten-Reshetikhin-Turaev invariant and analytic functions on roots of unity

15:20 - 16:20

Don Zagier (Max Planck Institute)

Modular properties of topological invariants and other q-series

### March 19

10:00 - 11:00

Jonathan Hillman (University of Sydney)

Indecomposable  $PD_3$ -complexes

11:20 - 12:20

Baptiste Morin (University of Bordeaux)

On the Weil-étale fundamental group of a number field

14:00 - 15:00

Ken-ichi Sugiyama (Chiba University)

On a geometric analog of Iwasawa conjecture

15:00 - 15:25

Walter Neumann (Columbia University)

Universal abelian covers in geometry and number theory

#### March 20

10:00 - 11:00

Thang Le (Georgia Institute of Technology)

Hyperbolic volumes, Mahler measure and homology growth

11:20 - 12:20

Shinya Harada (Kyushu University)

Hasse-Weil zeta function of absolutely irreducible  $SL_2$ -representations of the figure 8 knot group

14:00 - 15:00

Jinsung Park (Korea Institute for Advanced Study)

Eta invariant and Selberg zeta function of odd type for convex co-compact hyperbolic manifolds

15:20 - 16:20

Tomoyoshi Yoshida (Tokyo Institute of Technology) An asymptotic behavior of basis elements of conformal blocks of SU(2) WZW model in the large level

# Low dimensional topology and number theory II

March 15-18, 2010 The University of Tokyo, Tokyo

# Program

#### March 15

9:30 - 10:30

Eriko Hironaka (Florida State University)

Mapping classes with small dilatation

11:00 - 12:00

Jonathan Hillman (University of Sydney)

Embedding 3-manifolds with circle actions

13:20 - 14:20

Daniel Silver (University of South Alabama)

Twisted Alexander invariants

14:40 - 15:40

Takashi Hara (The University of Tokyo)

Reidemeister torsion, p-adic zeta function and its non-abelianization

16:00 - 17:00

Yoshikazu Yamaguchi (The University of Tokyo)

On the twisted Alexander polynomial for cyclic covers over knot exteriors

## March 16

9:30 - 10:30

Yoshiyuki Yokota (Tokyo Metropolitan University)

On the complex volumes of hyperbolic knots

11:00 - 12:00

Tudor Dimofte (California Institute of Technology)

TQFT and the Volume Conjecture

13:20 - 14:20

Hiroyuki Fuji (Nagoya University)

The Volume Conjecture and Topological Recursion

14:40 - 15:40

Toshie Takata (Niigata University)

On the SO(N) and Sp(N) free energy of a closed oriented 3-manifold

16:00 - 17:00

Susama Agarwala (California Institute of Technology)

The  $\beta$ -function over curved space-time under  $\zeta$ -function regularization

18:00

Banquet (UNOSATO)

#### March 17

9:30 - 10:30

Thang Le (Georgia Institute of Technology)

Integrality of the Witten-Reshetikhin-Turaev invariant of 3-manifolds and unified invariant for rational homology 3-spheres

11:00 - 12:00

Kazuo Habiro (RIMS, Kyoto University)

On the reduced colored Jones polynomials of links

13:20 - 14:20

Javier López Peña (Queen Mary University of London)

Cyclotomy and the field with one element

14:40 - 15:40

Sachiko Ohtani (National Defense Academy)

An Analogy between Deformation Spaces of Representations of Knot Groups and Galois Groups

16:00 - 17:00

Baptiste Morin (California Institute of Technology)

The Weil-étale topos and Deninger's dynamical system

### March 18

9:30 - 10:30

Ken-ichi Sugiyama (Chiba University)

On an analogy between number theory and hyperbolic geometry

11:00 - 12:00

Takahiro Kitayama (The University of Tokyo)

Non-commutative Reidemeister torsion and Morse-Novikov theory

13:20 - 14:20

Takayuki Morifuji (Tokyo University of Agriculture and Technology)

Twisted Alexander polynomials and nonabelian representations of 2-bridge knot groups

14:40 - 15:40

Susan Williams (University of South Alabama)

Alexander-Lin twisted polynomials

# Low dimensional topology and number theory III

March 14-17, 2011 Nishijin Plaza, Fukuoka

# Program

#### March 14

9:30 - 10:30

Daniel Silver (University of South Alabama)

Twisted Alexander polynomials of twisted knots

10:50 - 11:50

Takahiro Kitayama (The University of Tokyo)

Non-commutative Reidemeister torsion for homology cylinders

13:30 - 14:30

Susan Williams (University of South Alabama)

Bounding the degree and roots of twisted Alexander polynomials

14:50 - 15:50

Jean Raimbault (Institut de Jussieu)

Asymptotic behaviour of homology

16:10 - 17:10

Thang Le (Georgia Institute of Technology)

Integrality of the Witten-Reshetikhin-Turaev SU(2) invariants

## March 15

9:30 - 10:30

Adam Sikora (SUNY, Buffalo)

Idele class field theory for low dimensional topology

10:50 - 11:50

Norihiko Minami (Nagoya Institute of Technology)

On the Connes-Consani-Soulé type zeta function for  $F_1$ -schemes

13:30 - 14:30

Kazuhiro Hikami (Naruto University of Education)

Superconformal algebra, mock theta function, and Mathieu moonshine

14:50 - 15:50

Anton Mellit (University of Cologne)

Cell decompositions of Siegel modular spaces and Morse theory

16:10 - 17:10

Shinya Harada (Korea Institute for Advanced Study)

Hasse-Weil zeta functions of SL<sub>2</sub>-character varieties of some arithmetic hyperbolic 3-manifolds

18:00

Banquet

#### March 16

9:30 - 10:30

Kathleen Petersen (Florida State University)

Character Varieties of Some Families of 3-manifolds

10:50 - 11:50

Akishi Kato (The University of Tokyo)

Geometry of colored Jones polynomials

13:30 - 14:30

Kazuo Habiro (RIMS, Kyoto University)

Quantum fundamental groups and quantum representation varieties for 3-manifolds

14:50 - 15:50

Hidekazu Furusho (Nagoya University)

On Associators

16:10 - 17:10

Akihiro Tsuchiya (IPMU, The University of Tokyo)

# Conformal Field Theory and Quantum Group

## March 17

9:30 - 10:30

Stéphan Baseilhac (Université Montpellier 2)

Asymptotic rigidity of the quantum hyperbolic invariants

10:50 - 11:50

Jinseok Cho (Waseda University)

Optimistic limit of the colored Jones invariant

13:30 - 14:30

Hiroyuki Fuji (Nagoya University)

Surface Operator and Topological String

14:50 - 15:50

Takeshi Torii (Okayama University)

On the K(n)-local category

# Low dimensional topology and number theory IV

March 12-15, 2012 Kyushu University, Fukuoka

# Program

#### March 12

10:00 - 10:50

Makoto Ozawa (Komazawa University)

On the Neuwirth conjecture for knots (Joint work with J. Hyam Rubinstein)

11:00 - 11:50

Shinya Harada (Korea Institute for Advanced Study)

SL<sub>2</sub> character varieties and zeta functions of arithmetic two-bridge links

14:00 - 14:50

Toshitake Kohno (The University of Tokyo)

Quantum and homological representations of braid groups

15:00 - 15:25

Fumiya Amano (Kyushu University)

On Milnor primes

15:35 - 16:25

Hiroaki Nakamura (Okayama University)

On some arithmetic monodromy invariant for once punctured torus

## March 13

10:00 - 10:50

Kentaro Ihara (Max Planck Institute)

Iterated integral of modular forms

11:00 - 11:50

Thang Le (Georgia Institute of Technology)

On the stability of the colored Jones polynomial

14:00 - 14:50

Kazuo Habiro (RIMS, Kyoto University)

To be announced

15:00 - 15:50

Marc Culler (University of Illinois at Chicago)

Peripherally elliptic representations and character varieties

18:00

Banquet (Tamaru in Momochi)

#### March 14

10:00 - 10:50

Kazuhiro Ichihara (Nihon University)

Exceptional surgeries on Montesinos knots (Joint work with In Dae Jong)

11:00 - 11:50

Yoriko Kodani (Nara Women's University)

A sequence of new bridge indices for links each of which has a trivial knot component

14:00 - 14:50

Makoto Sakuma (Hiroshima University)

On Heckoid groups associated with 2-bridge links

15:00 - 15:25

Jun Ueki (Kyushu University)

On Iwasawa invariants for 3-manifolds

15:35 - 16:25

Joan Porti (Universitat Autonoma de Barcelona)

Reidemeister torsion and volume for hyperbolic knots

# March 15

10:00 – 10:50 Hidekazu Furusho (Nagoya University) On Galois action on knots

11:00 – 11:50 Tudor Dimofte (IAS, Princeton) Nonabelian Torsion and the Neumann-Zagier Equations

# Low dimensional topology and number theory V

March 11-14, 2013 Soft Research Park Center, Fukuoka

# Program

#### March 11

 $10:00 \sim 11:00$ 

Yasushi Mizusawa (Nagoya Institute of Technology) Iwasawa invariants and Greenberg type problem for links (a joint work with Teruhisa Kadokami)

 $11:10 \sim 12:00$ 

Jun Ueki (Kyushu University)

On Iwasawa invariants for links and "Kida's formula"

 $14:00 \sim 15:00$ 

Kathleen Petersen (Florida State University)

Character Varieties and Dehn Filling

 $15:10 \sim 16:10$ 

Takahiro Kitayama (The University of Tokyo)

On an analogue of Culler-Shalen theory for higher-dimensional representations

17:30 Bunquet (Italian at La Manina)

### March 12

 $10:00 \sim 11:00$ 

Ayako Ido (Nara Women's University)

Heegaard splitting with distance exactly n

(joint work with Yeonhee Jang and Tsuyoshi Kobayashi)

 $11:10 \sim 12:10$ 

Steve Boyer (Université du Quebec à Montréal)

L-spaces, left-orderability and foliations

 $14:00 \sim 15:00$ 

Atsushi Katsuda (Kyushu University)

Toward a geometric density theorem for nilpotent extensions

 $15:10 \sim 16:10$ 

Jinsung Park (Korea Institute for Advanced Study)

Bergman tau function and Chern-Simons invariant

### March 13

 $10:00 \sim 11:00$ 

Shunji Moriya (Kyoto University)

Homology of space of long knots and operad formality

 $11:10 \sim 12:10$ 

Kazuo Habiro (RIMS, Kyoto University)

Kirby calculus for null-homologous framed links in 3-manifolds

 $14:00 \sim 15:00$ 

Takefumi Nosaka (Kyushu University)

Longitudes in  $SL_2$ -representation spaces of knot groups and Milnor-Witt  $K_2$ -groups of fields

 $15:10 \sim 16:10$ 

Thang Le (Georgia Institute of Technology)

On the growth of torsions and regulators in finite abelian coverings

#### March 14

 $10:00 \sim 11:00$ 

Jun Murakami (Waseda University)

On the logarithmic knot invariants and the hyperbolic volume

 $11:10 \sim 12:10$ 

Rei Inoue (Chiba University)

Cluster Algebra and Complex Volume of Two-Bridge Knots

 $14:00 \sim 14:40$ Hirofumi Niibo (Kyushu University) Idèlic class field theory for 3-manifolds

 $14:50 \sim 15:50$  Gregor Masbaum (Institut de Mathématiques de Jussieu) Integral TQFT and modular representations of mapping class groups

# Low dimensional topology and number theory VI

March 18 - 21, 2014 Soft Research Park Center, Fukuoka

# Program

#### March 18

 $10:00 \sim 11:00$ 

Sakie Suzuki (Kyushu University)

The universal  $\mathfrak{sl}_2$  invariant and Milnor's invariants

 $11:15 \sim 12:15$ 

Jochen Gärtner (University of Heidelberg)

Massey products and mild pro-p fundamental groups

 $14:00 \sim 15:00$ 

Naotake Takao (RIMS, Kyoto University)

On the monodromy representation associated to the universal family of algebraic curves

 $15:15 \sim 16:15$ 

Hiroyuki Fuji (Tsinghua University)

The colored HOMFLY homology and super-A-polynomial

#### March 19

 $10:00 \sim 11:00$ 

Kirsten Wickelgren (Harvard University)

Massey products in Galois cohomology via rational points

 $11:15 \sim 12:15$ 

Shinya Harada (Tokyo Institute of Technology)

Hasse-Weil zeta functions of A-polynomials of torus knots

 $14:00 \sim 15:00$ 

Ido Efrat (Ben Gurion University)

Filtrations of absolute Galois groups

 $15:15 \sim 16:15$ 

Megumi Hashizume (Nara Women's University)

On the homomorphism induced by region crossing change

#### March 20

 $10:00 \sim 11:00$ 

Yuichiro Takeda (Kyushu University)

Higher arithmetic Chern character

 $11:15 \sim 12:15$ 

Thang Le (Georgia Institute of Technology)

On the skein modules of 3-manifolds at roots of unity

 $14:00 \sim 15:00$ 

Anna Beliakova (Universität Zürich)

Trace as an alternative decategorification functor

 $15:15 \sim 16:15$ 

Ken-ichi Sugiyama (Chiba University)

On a generalization of Deuring's results

#### March 21

 $10:00 \sim 11:00$ 

Akira Yasuhara (Tokyo Gakugei University)

Milnor Invariants and the HOMFLYPT Polynomial

 $11:15 \sim 12:15$ 

Yuriko Umemoto (Osaka City University)

The growth function of hyperbolic Coxeter dominoes and 2-Salem numbers

 $14:00 \sim 15:00$ 

Kazuo Habiro (RIMS, Kyoto University)

Kirby calculus for null-homologous framed links in 3-manifolds

#### Abstract

Anna Beliakova, Trace as an alternative decategorification functor

Categorification is a lifting of a given mathematical structure to a higher categorical level. Decategorification is the inverse process of simplifying higher structure into the original one. Both procedures are not unique. Usually, the Grothendieck group  $K_0$  is used as a decategorification functor. In this talk, we illustrate on the example of categorified quantum groups that the trace or 0th Hochschild homology is an interesting alternative to  $K_0$ . We show that duality between trace and center gives rise to an action of the current algebra Usl(n)[t] on the center of any 2-representation of the categorified quantum sl(n). This was previously observed by Brundan for t=1.

Hiroyuki Fuji, The colored HOMFLY homology and super-A-polynomial

I will discuss about the colored HOMFLY homology via string dualities. In recent years, the string dualites are applied to the quantum knot invariants, and some novel aspects of such invariants are studied explicitly in the physics language. One of the main focus of my talk is the physical realization of the categorification of the colored HOMFLY polynomial that is named as the colored HOMFLY homology. Utilizing string dualities, for some knots, we have obtained the explicit formula for the Poincaré polynomial of this homology which is known as the colored superpolynomial. From such explicit formulas, we further discovered the existence of the q-difference operator annihilating the colored superpolynomial for some knots, and it leads to a 2 parameter extension of the A-polynomial in the semi-classical limit  $q \to 1$ . We named this extended A-polynomial as super-A-polynomial. This talk is based on series of works in collaboration with S. Gukov, P. Sułkowski, and M. Stošić.

Jochen Gärtner, Massey products and mild pro-p fundamental groups

In this talk we investigate the étale pro-p fundamental group  $\pi_1^{\text{\'et}}(\operatorname{Spec} \mathcal{O}_k \setminus S)(p)$ , where S is a finite set of primes of the ring of integers  $\mathcal{O}_k$  of a number field k. In the first part we discuss arithmetic consequences of  $\operatorname{Spec} \mathcal{O}_k \setminus S$  being of type  $K(\pi; 1)$  for p due to A. Schmidt and show how the first explicit

examples in the difficult tame case have been obtained by J. Labute using the theory of mild pro-p groups. In the second part we recall the notion of Massey products in the cohomology of pro-p groups. We give an application of Massey products in order to construct mild pro-p groups and discuss arithmetic interpretations.

K. Wickelgren, Massey products in Galois cohomology via rational points

The Milnor conjecture identifies the cohomology ring  $H^*(Gal(\bar{k}/k), \mathbb{Z}/2\mathbb{Z})$  with the tensor algebra of  $k^*$  mod the ideal generated by  $x \otimes 1 - x$  for  $x \in k \setminus \{0,1\}$  mod 2. In particular,  $x \cup 1 - x$  vanishes, where x in  $k^*$  is identified with an element of  $H^1$ . We show that order n Massey products of n-1 factors of x and one factor of 1-x vanish by embedding  $\mathbb{P}^1 \setminus \{0,1,\infty\}$  into its Picard variety and constructing  $Gal(\bar{k}/k)$  equivariant maps from  $\pi_1^{\text{\'et}}$  applied to this embedding to unipotent matrix groups. This also identifies Massey products of the form  $\langle 1-x,x,?,x,1-x\rangle$  with  $f \cup 1-x$ , where f is a certain cohomology class which arises in the description of the action of  $Gal(\bar{k}/k)$  on  $\pi_1^{\text{\'et}}(\mathbb{P}^1 \setminus \{0,1,\infty\})$ .

# Low dimensional topology and number theory VII

March 25 - 28, 2015

IMAQ Seminar Room, Innovation Plaza, Momochihama, Fukuoka, JAPAN

## **Program**

## March 25 (Wednesday)

 $11:00 \sim 12:00$ 

Thang Le (Georgia Institute of Technology)

On the quantum trace map in the theory of quantum Teichmuller spaces

 $14:00 \sim 15:00$ 

Takahiro Matsushita (The University of Tokyo)

Box complexes and Kronecker double coverings of graphs

 $15:15 \sim 16:15$ 

Yuri Berest (Cornell University)

Double affine Hecke algebras and character varieties of knots

## March 26 (Thursday)

 $10:00 \sim 11:00$ 

Akishi Kato (The University of Tokyo)

Quiver mutation loops and partition q-series

 $11:15 \sim 12:15$ 

Irina Davydenkova (Université de Genève)

Inequalities from Poisson brackets

 $14:00 \sim 15:00$ 

Tetsuya Ito (RIMS, Kyoto University)

A topological description of colored Alexander invariant

 $15:15 \sim 16:15$ 

Gwénaël Massyeau (Universitè de Strasbourg & CNRS)

A functorial extension of the Reidemeister torsion to the category of (2+1)-dimensional cobordisms.

## Banquet

# March 27 (Friday)

 $10:00 \sim 11:00$ 

Kenji Sakugawa (Osaka Univeristy)

Polylogarithmic analogue of the Coleman-Ihara formula

 $11:15 \sim 12:15$ 

Anton Alekseev (Université de Genève)

The Grothendieck-Teichmueller group and the Kashiwara-Vergne problem

 $14:00 \sim 15:00$ 

Kazuo Habiro (RIMS, Kyoto University)

Traces of categorified quantum groups

 $15:15 \sim 16:15$ 

Kentaro Okamoto (Kyushu University)

Properties of braid zeta function

# March 28 (Saturday)

 $10:00 \sim 11:00$ 

Makoto Sakuma (Hiroshima University)

Mapping class group action on the space of geodesic rays

 $11:15 \sim 12:15$ 

Tsuyoshi Kobayashi (Nara Women's University)

A construction of flat-foldable origami via similarity structure

### Abstract

Anton Alekseev (Université de Genève)

The Grothendieck-Teichmueller group and the Kashiwara-Vergne problem In this talk, we will explain the relation between the Kashiwara-Vergne problem in Lie theory and the Grothendieck-Teichmueller group. The Kashiwara-Vergne problem is stated in terms of the Campbell-Hausdorff series and a certain equation for cyclic words in 2 variables. The Grothendieck-Teichmueller group gives rise to symmetries of the Kashiwara-Vergne problem. This observation leads to an interesting conjecture about the Grothendieck-Teichmueller Lie algebra.

The talk is based on joint works with C. Torossian.

Yuri Berest (Cornell University)

Double affine Hecke algebras and character varieties of knots

Let G be a complex reductive algebraic group. In this talk, we will discuss a general conjecture that there is a canonical action of a double affine Hecke algebra of type G on a (quantized) character variety of a knot complement in  $S^3$ . We will give some motivation and present an explicit construction in the rank one case. We then discuss some examples and implications of our conjecture. As a main application, we construct 3-variable polynomial knot invariants that specialize to the classical colored Jones polynomials introduced by Witten, Reshetikhin and Turaev.

The talk is based on joint work with P. Samuelson.

Irina Davydenkova (Université de Genève)

Inequalities from Poisson brackets

We introduce the notion of tropicalization for Poisson structures on  $\mathbb{R}^n$  with coefficients in Laurent polynomials. To such a Poisson structure we associate a polyhedral cone and a constant Poisson bracket on this cone. There is a version of this formalism applicable to  $\mathbb{C}^n$  viewed as a real Poisson manifold. In this case, the tropicalization gives rise to a completely integrable system with action variables taking values in a polyhedral cone and angle variables spanning a torus.

As an example, we consider the canonical Poisson bracket on the dual Poisson-Lie group  $G^*$  for G = U(n) in the cluster coordinates of Fomin-Zelevinsky defined by a certain choice of solid minors. We prove that the corresponding integrable system is isomorphic to the Gelfand-Zeiltin completely integrable system.

Kazuo Habiro (RIMS, Kyoto University)

Traces of categorified quantum groups

The trace (or cocenter of the 0th Hochschild-Mitchell homology) of a category is the space of endomorphisms modulo the relation fg = gf. The notion of trace can be naturally applied to categories with structures. For example, the trace of a monoidal category is a monoid and the trace of a 2-category is a (1-)category.

I plan to talk about generalities about trace and then the traces of categorified quantum groups.

Tetsuya Ito (RIMS, Kyoto University)

A topological description of colored Alexander invariant

The Alexander polynomial (invariant) is one of the most basic invariant of knots, and plays an important role in arithmetic topology. In this talk I present a homological description of the colored Alexander invariant, a quantum-invariant theoretical generalization of Alexander invariant. Our description suggests a connection between an abelian covering of (the configuration space of) the knot complement and partially explains that colored Alexander invariant is indeed a generalization of the Alexander polynomial, in a topological point of view.

Akishi Kato (The University of Tokyo)

Quiver mutation loops and partition q-series

This is a joint work with Yuji Terashima (Tokyo Institute of Technology); arXiv:1403.6569 & arXiv:1408.0444, Comm. Math. Phys. to appear.

Quivers (oriented graphs) and their mutations are ubiquitous in mathematics and mathematical physics. They play a key role in wall-crossing phenomena, gluing of ideal tetrahedra, etc.

Recently, we introduced a partition q-series  $Z(\gamma)$  for a quiver mutation loop  $\gamma$  (a loop in a quiver exchange graph in cluster algebra terminology). This enjoys following remarkable properties: (1)  $Z(\gamma)$  is invariant under "inversion" and "cyclic shift" of  $\gamma$ ; so it may be regarded as a monodromy invariant. (2)  $Z(\gamma)$  satisfies pentagon identities. (3) If the quivers are of Dynkin type or square products thereof, they reproduce so-called fermionic character formulas of certain modules associated with affine Lie algebras; in particular they

have nice modular properties. (4) If the mutation sequences  $\gamma$  is reddening,  $Z(\gamma)$  is expressed as a product of quantum dilogarithms and coincides with the combinatorial Donaldson-Thomas invariant.

The definition of  $Z(\gamma)$  requires only combinatorial data of quivers and mutation loops, and completely independent of the details of the models behind. It is hoped that a deeper understanding of the partition q-series shed new lights on dualities and quantization.

Tsuvoshi Kobayashi (Nara Women's University)

A construction of flat-foldable origami via similarity structure

Origami is a Japanese traditional artwork made of a square paper, and is studied by many researchers, not only because it is theoretically interesting but also because it has lots of practical applications. In this talk, I will describe a method for producing flat-foldable origami by using similarity structure on 2-dimensional torus, and Lorenz template. I will also mention some topics related them.

Thang Le (Georgia Institute of Technology)

On the quantum trace map in the theory of quantum Teichmuller spaces

Takahiro Matsushita (The University of Tokyo)

Box complexes and Kronecker double coverings of graphs

Coloring of a graph is to assign a color to each vertex so that adjacent vertices have different colors. The chromatic number is the smallest number of colors we need to color a given graph. The graph coloring problem is to determine the chromatic number, and is one of the oldest problem in graph theory.

The box complex is a  $Z_2$ -space assigned to a graph, and its topology is known to be closely related with the chromatic number. In this talk I related box complexes with the canonical double coverings over graphs, called the Kronecker double coverings.

The main result states that if the Kronecker double coverings of two graphs coincide then their box complexes are isomorphic. As an application, I constructed graphs G, H such that their box complexes are isomorphic but their chromatic numbers are different.

Gwénaël Massyeau (Universitè de Strasbourg CNRS)

A functorial extension of the Reidemeister torsion to the category of (2+1)-dimensional cobordisms.

(Joint work with Vincent Florens.) In 3-dimensional topology, the Reidemeister torsion is a classical invariant which usually applies to manifolds with empty or toroidal boundary. In this talk, we will consider Reidemeister torsion for 3-manifolds with arbitrary boundary. Given a commutative field F and a multiplicative subgroup G of F, we will construct in this way a functor from the category of (2+1)-dimensional cobordisms equipped with G-representations to the category of graded F-vector spaces. We will show a few properties of this functor and how it specializes to some well-known Alexander-type invariants

Kentaro Okamoto (Kyushu University)

Properties of braid zeta function

There is a well-known zeta function of an element of the symmetric group as a toy model.

In this study, considering the extension from the symmetric group to the braid group we introduce a new zeta function of braids by using the Burau representation. We will show that this braid zeta function satisfies a functional equation and the analogue of Riemann Hypothesis under a certain assumption, and that the Alexander polynomial is contained in the residue of this braid zeta function at s=1. Furthermore, we define a special product as an operation of braids, and show a "decomposition theorem" of the zeta function of a braid which is represented by special products. This theorem leads to the decomposition formula of the Alexander polynomial of a special knot as a corollary.

Kenji Sakugawa (Osaka Univeristy)

Polylogarithmic analogue of the Coleman-Ihara formula

Abstract: Let K be a number field, z an element of K, p an odd prime and m a positive integer. The value  $\ell i_m(z)$  of the  $\ell$ -adic polylogarithm of weight m at z is a continuous map from the absolute Galois group  $G_K$  of K to the  $G_K$ -module  $\mathbb{Q}_p(m)$ . In this talk, we give a sufficient condition for the linear sum  $\sum_i a_i \ell i_m(z_i)$ ,  $a_i \in \mathbb{Z}$ ,  $z_i \in K$  being a 1-cocycle. Then, we give a formula relating the restriction of that linear sum to a inertia subgroup of  $G_K$  at a place v over p and Coleman's p-adic polylogarithms. This is joint work with Hiroaki Nakamura and Zdzislaw Wojtkowiak.

Makoto Sakuma (Hiroshima University)

Mapping class group action on the space of geodesic rays of a punctured

hyperbolic surface

For a hyperbolic punctured surface S of finite area, consider the space, G, of geodesic rays emanating from punctures. Note that G is identified with the disjoint union of the horo-cycles around the punctures and that the mapping class group of S naturally acts on the space G.

In the first half of my talk, I will explain the role of the action in the variations of McShane's identity (i) for punctured surface bundles (Bowditch and Akiyoshi-Miyachi-Sakuma) and (ii) for 2-bridge links (Lee-Sakuma).

In the second half of my talk, I will explain the following theorem proved by Bowditch, answering to a my question.

Theorem. The non-wandering set of the action of the mapping class group on the space G has measure 0.

I would also like to discuss the mapping class group action on the space of "simple" geodesic rays.

# Low dimensional topology and number theory VIII

March 22 - 25, 2016

IMAQ Seminar Room, Innovation Plaza, Momochihama, Fukuoka, JAPAN

## **Program**

## March 22 (Tuesday)

 $9:20 \sim 10:10$ 

Kazuo Habiro (Kyoto University, RIMS)

On the category of finitely generated free groups

 $10:25 \sim 11:15$ 

Thang Le (Georgia Institute of Technology)

On the q-holonomicity of the colored HOMFLYPT polynomial

 $11:30 \sim 11:55$ 

Kentaro Okamoto (Kyushu University)

The zeta function and Jones polynomial

 $14:30 \sim 15:20$ 

Ted Chinburg (University of Pennsylvania)

Negative curves on surfaces and hyperbolic codes

 $15:40 \sim 16:30$ 

Yasushi Mizusawa (Nagoya Institute of Technology)

Tame pro-p Galois groups over cyclotomic  $\mathbb{Z}_p$ -extensions

### March 23 (Wenesday)

 $9:20 \sim 10:10$ 

Tomoki Mihara (University of Tokyo)

On M<sup>2</sup>KR dictionary for metrised 3-dimensional manifolds

 $10:25 \sim 11:15$ 

Nigel Boston (University of Wisconsin-Madison)

Moments of distributions of algebraic fundamental groups

 $11:30 \sim 11:55$ 

Hisatoshi Kodani (Kyushu University)

Group-like expansions, string links and some remarks on arithmetic topology

 $14:30 \sim 15:20$ 

Hiroshi Tsunogai (Sophia Univeristy)

Galois orbits of genus 1 dessins of degree up to 6

 $15:40 \sim 16:30$ 

Herbert Gangl (Durham University)

Zagier's polylogarithm conjecture revisited

Banquet

# March 24 (Thursday)

 $9:20 \sim 10:10$ 

Shinya Harada (Tokyo Institute of Technology)

Hasse-Weil zeta functions of  $SL_2$  (PSL<sub>2</sub>)-character varieties of closed orientable hyperbolic 3-manifolds

 $10:25 \sim 11:15$ 

Adam Sikora (State University of New York at Buffalo)

Algebraic Properties of Skein Algebras of Surfaces

 $11:30 \sim 11:55$ 

Ryoto Tange (Kyushu University)

On certain L-functions for deformations of knot group representations

 $14:30 \sim 15:20$ 

Takao Satoh (Tokyo University of Science)

On the  $\mathrm{SL}(m,\mathbb{C})$ -representation rings of free groups and the Johnson homomorphisms

 $15:40 \sim 16:30$ 

Anh Tran (The University of Texas at Dallas)

The topology of the Jones polynomial

March 25 (Friday)

 $9:20 \sim 10:10$ 

Yu Iijima (Kyoto University, RIMS) Galois action on the image of the pro-l universal monodromy representation of the moduli stack of hyperbolic curves

 $10:25 \sim 11:15$ 

Hidekazu Furusho (Nagoya University) Complex and p-adic multiple zeta functions

 $14:00 \sim 14:50$ 

Nobuyoshi Takahashi (Hiroshima University) Quandles associated to arithmetic schemes

 $15:10 \sim 16:00$ 

Alan Reid (The University of Texas at Austin) Azumaya algebras and canonical components

#### Abstract

Nigel Boston (University of Wisconsin-Madison)

Moments of distributions of algebraic fundamental groups

There are various important conjectures on distributions of Galois groups of unramified extensions of imaginary quadratic fields. In this talk I shall explain how these are more conveniently described in terms of moments and prove some function field analogues. Joint work with Melanie Wood and others.

Ted Chinburg (University of Pennsylvania)

Negative curves on surfaces and hyperbolic codes

This talk is about joint work with Matt Stover on curves of small genus and negative self intersection on a smooth projective surface X. We show that there are only finitely many such curves having arithmetic genus less than one-fourth the first Betti number of the surface; this result is sharp over finite fields. In the Neron Severi group of the surface, the classes of the above curves form a counterpart in hyperbolic space of a spherical code. By bounding the number of points in such hyperbolic codes, we bound the number of curves of the above kind in terms of the rank of the Neron Severi group of the surface.

Hidekazu Furusho (Nagoya University)

Complex and p-adic multiple zeta functions

This talk is based on my joint work with Y. Komori, K. Matsumoto and H. Tsumura. In the first half of my talk, I will introduce our method of desingularization of multiple zeta-functions. I will explain that multiple zeta-functions (which are known to be meromorphic with infinitely many singular loci in the whole space) turn to be entire by our integral method. I will introduce various properties of our methods: particularly I will reveal that the desingularized function is given by a suitable finite 'linear' combination of multiple zeta-functions with some arguments shifted. I will also explain that specific combinations of Bernoulli numbers attain the special values at their non-positive integers of the desingularized ones. Based on them, in the second half, I will explain our construction of (several variable) p-adic multiple L-functions which generalizes that of Kubota-Leopoldt's p-adic L-functions. Then I will show their various fundamental properties including

multiple Kummer congruences, functional relations as well as their evaluations at both positive and negative integer points.

Herbert Gangl (Durham University)

Zagier's polylogarithm conjecture revisited

Abstract: In the early nineties, Goncharov proved the weight 3 case of Zagier's Conjecture stating that the special value  $\zeta_F(3)$  of a number field F is essentially expressed as a determinant of trilogarithm values taken in that field. He also envisioned a vast–partly conjectural–programme of how to approach the conjecture for higher weight. We can remove one important obstacle in weight 4 by solving one of Goncharov's conjectures. It further allows us to deduce a functional equation for  $Li_4$  in four variables as one expects to enter in a more explicit definition of a certain algebraic K-group.

Kazuo Habiro (Kyoto University, RIMS) On the category of finitely generated free groups

I plan to give an elementary proof of the well-known fact that the category of finitely generated free groups is equivalent as a symmetric monoidal category to the free symmetric monoidal category generated by a commutative Hopf monoid. I plan to discuss also some related topics.

Shinya Harada (Tokyo Institute of Technology)

Hasse-Weil zeta functions of  $SL_2$  (PSL<sub>2</sub>)-character varieties of closed orientable hyperbolic 3-manifolds

The  $SL_2(\mathbb{C})$ -character variety of a 3-manifold plays an important role in the study of 3-dimensional topology, which is known to be an algebraic set over the rational number field. For an orientable hyperbolic 3-manifold of finite volume we have an associated holonomy representation and there is an irreducible component of the  $SL_2(\mathbb{C})$ -character variety containing the character of a lift of the holonomy representation (called a canonical component). In this talk we will consider the closed hyperbolic 3-manifold case and explain that the Hasse-Weil zeta function of the canonical component of the  $SL_2$ -character variety ( $PSL_2$ -character variety in the sense of Heusener-Porti) of a closed orientable hyperbolic 3-manifold of finite volume is equal to the Dedekind zeta function of the trace field (invariant trace field). Here the trace field is a number field generated by the trace of the image of the holonomy representation and the invariant trace field is a subfield of the trace field

generated by the trace of the square of the image. Especially we can see that when the closed 3-manifold is an arithmetic 3-manifold, the special value at s=2 of the zeta function of the PSL<sub>2</sub>-character variety is expressed in terms of the hyperbolic volume of the manifold.

### Yu Iijima (Kyoto University, RIMS)

Galois action on the image of the pro-l universal monodromy representation of the moduli stack of hyperbolic curves

By considering the outer action of the étale fundamental group of the moduli stack of hyperbolic curves over  $\overline{\mathbb{Q}}$  on the pro-l fundamental group of a fiber of the universal curve over this moduli stack, we may obtain the pro-l universal monodromy representation of the moduli stack of hyperbolic curves. This representation may be regarded as a pro-l version of the natural outer action of the mapping class group on the topological fundamental group of a connected orientable compact topological surface. The absolute Galois group of  $\mathbb Q$  acts naturally on the image of this representation up to inner automorphisms. In this talk, we study the structure of the image of the pro-l universal monodromy representation by using the outer Galois action on this image.

# Hisatoshi Kodani (Kyushu University)

Group-like expansions, string links and some remarks on arithmetic topology In this talk, I introduce the notion of normalized expansions which satisfies a boundary condition of a punctured disk and I give some applications of these expansions in the study of string links. When normalized expansions are special expansions, we have the special Artin representation which gives the special longitudes of a string link. I will also explain that the coefficients of a special longitude give a topological analogue of multiple zeta values in some sense.

Thang Le (Georgia Institute of Technology) On the q-holonomicity of the colored HOMFLYPT polynomial

Tomoki Mihara (The University of Tokyo)

On M<sup>2</sup>KR dictionary for metrised 3-dimensional manifolds

I develop M<sup>2</sup>KR dictionary of arithmetic topology for branched Abelian coverings of metrised 3-dimensional manifolds such as 3-dimensional Riemannian manifolds. I study analogy between rectifiable knots in metrised closed ori-

ented 3-dimensional rational homology spheres and prime ideals of number fields. Since the class of rectifiable paths contains the class of geodesic paths, the class of recitifiable knots generates the first homology group, and hence possesses sufficient data on Abelian coverings. The arc-length of rectifiable knots corresponds to the cardinality of the residue fields, and the pairing given by linking numbers yields an analogue of the notion of the reduction to the residue field. This correspondence is not just an extension of the analogy between linking numbers modulo 2 and Artin symbols in class field theory for number fields, and plays an important role in the observation of local-global compatibility of class field theory for manifolds.

Yasushi Mizusawa (Nagoya Institute of Technology)

Tame pro-p Galois groups over cyclotomic  $\mathbb{Z}_p$ -extensions

For a prime number p and a number field, we consider the maximal pro-p-extension with restricted ramification, which is tamely ramified over the cyclotomic  $\mathbb{Z}_p$ -extension. From a view of arithmetic topology, the Galois groups of such extensions seem to be analogous to link groups, and  $\mathbb{Z}_p$ -extensions are analogous to branched  $\mathbb{Z}$ -covers of a link. An essential part of such a Galois group is the Galois subgroup corresponding to the tamely ramified extension over the  $\mathbb{Z}_p$ -extension. This tame pro-p Galois group has been studied in Iwasawa theory with the  $\mathbb{Z}_p$ -action on it. In this talk, I will explain some explicit examples of these Galois groups with some related topics.

Kentaro Okamoto (Kyushu University)

The zeta function and Jones polynomial

There is the well known zeta function of the dynamical system generated by an element of the symmetric group. By considering this zeta function as a model, we construct the braid zeta function by using the Burau representation. Then the Alexander polynomial of the closure of a braid is interpreted as the residue of this braid zeta function. Furthermore, we also construct the zeta function by using the Jones representation of the braid group. In this workshop, we talk about the relation between the zeta function and the Jones polynomial.

Alan Reid (The University of Texas at Austin)

Azumaya algebras and canonical components

Abstract: Suppose that G is the fundamental group of a finite volume orientable 1-cusped hyperbolic 3-manifold M. Thurston identified a particular

curve component C (the canonical component) of the character variety. This component contains the faithful discrete character as well as those characters arising from Dehn surgery. In this talk we will describe how one obtains a so-called Azumaya algebra, A over a Zariski-open subset of C and derive conditions on when A extends over all points of the smooth projective model of C. As a consequence we will see how this puts significant restrictions on arithmetic invariants of Dehn surgeries on M. In addition, when M is the complement of a hyperbolic knot K in  $S^3$ , we can describe a sufficient condition for A to extend over C in terms of the Alexander polynomial of K.

Takao Satoh (Tokyo University of Science)

On the  $\mathrm{SL}(m,\mathbb{C})$ -representation rings of free groups and the Johnson homomorphisms

First, we recall some facts about the Johnson homomorphisms and twisted first cohomology groups of the automorphism groups of free groups. Then we introduce the rings of  $SL(2,\mathbb{C})$ -representations of free groups. By using it, we can construct analogs of the Johnson homomorphisms. In this talk, we show that the first homomorphism can be extended to the automorphism group of a free group as a crossed homomorphism, and show that it induces Kawazumi's 1-cocycle and Morita's 1-cocycle of the automorphism group of a free group.

Adam Sikora (State University of New York at Buffalo)

Algebraic Properties of Skein Algebras of Surfaces

For a surface F, the space of links in  $F \times [0,1]$  modulo Kauffman bracket skein relations is called the skein algebra of F, denoted by S(F). It is a non-commutative deformation of the  $SL(2,\mathbb{C})$ -character variety of F, of significant importance to quantum topology. In particular, for F with boundary, it is (almost) the quantum Teichmuller space of F. Except for a few simplest surfaces F, not much is known about the algebraic properties of S(F) for closed F. We are going to explore some of their two fundamental properties.

Nobuyoshi Takahashi (Hiroshima University)

Quandles associated to arithmetic schemes

A quandle is an algebraic system defined by a binary operation. To a knot K, its "knot quandle" Q(K) is associated, and Q(K) determines K up to equivalence. After a brief introduction to quandles, I will talk about a certain analogue of the knot quandle in number theory: For a normal, separated and

integral scheme X of finite type over  $\mathbb{Z}$  and a set  $\mathcal{M}$  of closed points of X, a topological quandle  $Q(X, \mathcal{M})$  is defined. Then I will discuss the problem of reconstruction. Let  $X = \operatorname{Spec} \mathcal{O}_K \setminus \{\mathfrak{p}\}$  where K is  $\mathbb{Q}$  or a quadratic field. Assume that  $\pi_1(X)^{\operatorname{ab}}$  is infinite, and that  $\mathcal{M}$  is of density 1. Then we can recover pretty much information about X and  $\mathcal{M}$  from  $Q(X, \mathcal{M})$  using results from p-adic transcendental number theory.

### Ryoto Tange (Kyushu University)

On certain L-functions for deformations of knot group representations. We study the twisted knot module for the universal deformation of an SL(2)-representation of a knot group, and introduce an associated L-function, which may be seen as an analogue of the algebraic p-adic L-function associated to the Selmer module for the universal deformation of a Galois representation. We then investigate two problems proposed by Mazur: Firstly we show the torsion property of the twisted knot module over the universal deformation ring under certain conditions. Secondly we verify the simplicity of the zeroes of the L-function by some concrete examples for 2-bridge knots. This is the joint work with T. Kitayama, M. Morishita and Y. Terashima.

Anh Tran (The University of Texas at Dallas)

The topology of the Jones polynomial

I will discuss old and new conjectures about the topology of the Jones polynomial. These include the AJ conjecture, the slope conjecture, and the strong slope conjecture. The AJ conjecture of Garoufalidis relates the A-polynomial and the colored Jones polynomial of a knot. The A-polynomial was introduced by Cooper et al. in 1994 and has been fundamental in geometric topology. A similar conjecture to the AJ conjecture was also proposed by Gukov from the viewpoint of the Chern-Simons theory. The slope conjecture of Garoufalidis and two new conjectures of Kalfagianni and myself are about the relationship between the degree of the colored Jones polynomial of a knot and the topology of the knot. These conjectures assert that certain boundary slopes and Euler characteristics of essential surfaces in a knot complement can be read off from the degree of the colored Jones polynomial.

Hiroshi Tsunogai (Sophia University)

Galois orbits of genus 1 dessins of degree up to 6

We calculated the defining equations of all Belyi pairs of genus 1 of degree up to 6, and determined the Galois orbits in these cases. As a result, we show that these Galois orbits can be separated by known Galois invariants of dessins: valency lists, monodromy groups, Nielsen classes, cartographic groups and automorphism groups. We will give some examples of the computation of such Belyi pairs.